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## Industry Differences in the Firm Size Distribution

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## Abstract

This paper empirically examines industry determinants of the shape of Swedish firm size distributions at the 3-digit (NACE) industry level between 1999-2004 for surviving firms. Recent theoretical studies have begun to develop a better understanding of the causal mechanisms behind the shape of firm size distributions. At the same time there is a growing need for more systematic empirical research. This paper therefore presents a two-stage empirical model, in which the shape parameters of the size distribution are estimated in a first stage, with firm size measured as number of employees. In a second stage regression analysis, a number of hypotheses regarding economic variables that may determine the distributional shape are tested. The result from the first step are largely consistent with previous statistical findings confirming a power law. The main finding, however, is that increases in industry capital and financial constraint exert a considerable influence on the size distribution, shaping it over time towards thinner tails, and hence fewer large firms.

**Keywords:** Firm size distribution · Zipf's law · Gibrat's law

**JEL classification:** L11 · L25 · D22

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## 1 Introduction

In the wake of studies by Robert Gibrat (1931) and research by Herbert Simon, who published a series of papers on the subject in the 1950s and 1960s, it is now generally accepted that firm size distributions is right skewed with a possible power law tail for larger firms (e.g. Growiec et al, 2008).

Recent applied research shows that the firm size distribution (often above a certain minimum size) tends to conform to a particular power law that is known as Zipf's law (Axtell, 2001; Gaffeo et al, 2003; Fujiwara et al, 2004; Okuyama et al, 1999). Named after the Harvard linguist George Zipf (1936), and originally used to describe the frequency of words in a natural language, the law when applied to firm sizes states that the frequency of a firm's size is inversely proportional to that size, i.e. a power law with exponent equal to minus one.

Working from census data, Axtell (2001) found that the firm size distribution for the entire population of US firms is consistent with Zipf's law. Similar results have been reported for a number of European countries by Fujiwara et al (2004), and further validated for the income distribution among Japanese firms by Okuyama et al (1999).

The fascination of this type of statistical invariants can be traced back to Schumpeter (1949), who in discussing Pareto's law for income distributions remarked that "they might lay the foundations for an entirely novel type of theory" (Schumpeter, 1949, p.155; quoted in Gabaix, 2009). Schumpeter's prediction has yet to be realized, but he identified two areas of inquiry that have dictated the discourse on the subject ever since. The first area of inquiry relates to the question of fit, i.e., the need to test whether the purported "invariants" hold, which has been a common focus in the aforementioned studies on the relationship between the firm size distribution and Zipf's law. The second area of inquiry concerns the ramifications if the law is indeed found to be valid. As Schumpeter asked: "What are we to infer from this?" (Schumpeter, 1949, p.155). This is the motivation of this paper.

Despite the demonstrated ubiquity of power laws and of Zipf's law in the firm size distribution across countries and over time, there remains no clear consensus as regards the interpretation of the accumulating empirical evidence. One reason why such an understanding is important is that the law may have implications for public policy (Axtell, 2001). For instance, power laws relate to concentration measures used by the United States Department of Justice and

the Federal Trade Commission to screen for undue competition in the event of mergers.

In the more statistically oriented literature, stochastic models and variations of Gibrat's law are often invoked as explanations of the right skewed shape of the firm size distribution (Simon and Bonini, 1958; Sutton, 1997; Gabaix, 1999; Saichev et al, 2009). Gibrat's law simply states that firm growth is independent of firm size and hence that firm size is scale independent rather than mean reverting. These explanations are interesting in their own right but are not based on economic variables, which makes policy recommendations difficult (Axtell, 2001). Nevertheless, this research does have some broad implications; for instance, a rejection of Zipf's law might justify economic policy intended to stimulate employment by encouraging the birth and growth of small firms.<sup>1</sup>

There is also a growing body of theoretical literature intent on exploring the economic mechanisms governing the firm size distribution, and enhancing our understanding of this phenomenon, as visualized by Schumpeter (1949). For instance, Cabral and Mata (2003) argue that financially constrained small and young firms cause a right skewed distribution with fewer large firms. In Rossi-Hansberg et al (2007), the shape of the industry firm size distribution is related to the intensity of industry-specific human capital. In particular, the firm size distribution has thinner tails in industries with a small share of human capital, such as the manufacturing industry.

Given the robust empirical evidence in support of power laws and the somewhat diffuse body of theoretical literature, there is a pressing need for more systematic empirical work about the determinants of the shape of the firm size distribution. The purpose of this paper is to address this gap. To address both of Schumpeter's inquiries, this paper draws from the statistical literature to construct an empirical model for industry-level firm size distributions that makes it possible to test a number of theoretical determinants hypothesized to shape these distributions.

The empirical strategy is inspired by Ioannides et al (2008), Rosen and Resnick (1980) and Soo (2005), who all construct two-stage empirical models to study the formation of city size distributions. The setting for the firm size distribution is analogous. First, the shape parameters of the firm size distri-

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<sup>1</sup>Cordoba (2008) demonstrated that there is an equivalence relationship between Gibrat's law and Zipf's law. As Wagner (1992) suggests, if Gibrat's law holds then regional stimulus based on firm size alone has little merit. Should instead Gibrat's law (Zipf's law) be rejected from finding small firms to grow faster, then policy programs centered on firm size might be justified.

bution are estimated across Swedish 3-digit industries according to the NACE (rev. 1) classification, using a technique outlined in Clauset et al. (2007; 2009). The resulting cross-industry variation is then explained in a second stage by regressing the parameter estimates onto a number of explanatory variables.

The model is tested on the industry-level firm size distribution in the period 1999-2004, using Swedish data on incorporated surviving firms active over the period 1997-2004. The main findings confirm that capital intensity and financial constraints both have a thinning effect on the tail of the firm size distribution. This finding has important policy implications because it suggests that improving firms' access to capital markets will result in a firm size distribution with a thicker tail, potentially allowing small firms to reach their optimal size, as advanced in Cabral and Mata (2003).

The remainder of the paper is structured as follows. Section 2 discusses related theoretical and empirical literature and presents the hypotheses that will be tested. In Section 3, the data are described along with the method used to test each hypothesis. Section 4 presents the results, and finally Section 5 identifies further implications of those results for economic policy and concludes the paper.

## **2 Theoretical background**

This section consists of two subsections. In the first, I discuss the relationship between Gibrat's law and the firm size distribution. In the second, I formulate hypotheses regarding economic variables that may determine the shape of the firm size distribution.

### **2.1 Gibrat's law and its implications for the firm size distribution**

Since the seminal works of Gibrat (1931), Simon and Bonini (1958) and Ijiri and Simon (1967), Gibrat's law has been examined and tested vigorously. Recent empirical research, however, finds little evidence that supports the strong version of the law, namely that firm size is statistically independent of firm growth (Hall, 1987; Evans, 1987a,b; Dunne et al, 1989; Dunne and Hughes, 1994; Audretsch et al, 2004; Calvo, 2006).

In fact, Mansfield (1962) demonstrated that there are good reasons to expect small firms to grow faster than large firms. For small firms to survive in industries characterized by a high minimum efficiency scale (MES), they must quickly reach a sufficient size to produce at minimum long-term average costs

(Audretsch et al, 2004). Mansfield (1962) therefore proposed a version of the law that only applies to firms that are larger than the industry MES. For this weaker version of the law, there exists some evidence of scale independence (Mowery, 1983; Hart and Oulton, 1996; Becchetti and Trovato, 2002; Lotti et al, 2003).

The evidence of a size boundary has theoretical and inferential implications for the firm size distribution. In fact, Gabaix (1999) showed that if firm (log) size is subject to a lower reflecting boundary (a bounded random walk), the firm size distribution takes the form of a power law instead of a lognormal distribution under Gibrat’s strong law. In the limiting case, as the boundary approaches zero, the distribution approaches Zipf’s law (Gabaix, 1999). A reflecting boundary forces firms to reach a sufficiently large size. Should that boundary coincide with the industry MES, this concept will be similar to Mansfield’s (1962) argument.

The notion of a lower boundary also maps into the definition of the power law. Confined to a positive interval  $[size_{min}, \infty)$ , a power law can be defined via its counter cumulative distribution function as follows:

$$P(size_{i,t} > x) = (x/size_{min})^{-\zeta}, \quad (1)$$

where the parameter  $size_{min} > 0$  constitutes a lower size boundary for some firm  $i$  at a time  $t$ . Knowledge of this boundary is necessary to avoid biased estimates, which is further discussed in Section 3.2. The exponent  $\zeta \in (0, \infty]$  determines the shape of the distribution; a small value reflects the existence of a thicker tail, and hence a higher probability that very large firms exist.<sup>2</sup> The exponent also alludes to the existence of different competitive regimes, as it relates to measures of industry dispersion and concentration that are of particular interest to policymakers and antitrust lawyers (Axtell, 2001).

Even if aggregate firm size distributions have been shown to follow a power law (Axtell, 2001; Gaffeo et al, 2003; Fujiwara et al, 2004; Okuyama et al, 1999; Cirillo and Husler, 2009), the distributions of industry cross-sections at a finer level of aggregation sometimes do not (Axtell et al, 2006). At the 4 and 5-digit NACE level, distributions often exhibit both substantially thinner

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<sup>2</sup>Because of its heavy tail, a power law differs from many other probability distributions in terms of its moments. If, e.g.,  $\zeta < 1$ , then the first moment is undefined. For  $1 < \zeta < 2$ , the expected value is well defined, but the second moment is not. Generally, for higher moments  $m$ ,  $\zeta > m$  must be satisfied for the  $m$ -th moment  $\langle size^m \rangle$  to exist and can then be calculated using  $\langle size^m \rangle = size_{min} \zeta / (\zeta - m)$ . The largest probable value can nevertheless always be calculated using  $\langle x_{max} \rangle \sim n^{1/\zeta}$ , where  $n$  is the number of observations for which  $size_{i,t} > size_{min}$  (Newman, 2005).

tails and multi-modality (Quandt, 1966). Dosi and Nelson (2010) speculate that bi-modality in the size distribution may reflect oligopolistic elements in an industry that separate core firms from fringe firms. One issue in the study of the firm size distribution across industries is therefore the industry scaling puzzle (Quandt, 1966; Dosi et al, 1995; Axtell et al, 2006; Dosi, 2007). Dosi (2007) argues that the aggregate power law results from aggregating heterogeneous manufacturing industries with several technology regimes that are associated with different interaction and learning processes. Axtell et al (2006) present a related argument: that industry-specific shocks result in deviations from a power law at the industry level but not at the aggregate level.

Some of the literature on Gibrat's law and the firm size distribution is mainly statistical in origin, bordering on econophysics, and includes little underlying economic modeling. This might be problematic given the role of power laws as attractors in generalized versions of the central limit theorem (Willinger et al, 2004; Feller, 1971). In sampling from various heavy-tailed distributions almost any mix or combination is likely to have a power law at the limit (Stumpf and Porter, 2012; Jessen and Mikosch, 2006; Willinger et al, 2004). Merely fitting data to a power law thus makes it difficult to determine whether the emerging shape is a result of mere aggregation, as speculated by Dosi (2007), or whether it reflects some economic mechanism. This fact motivates the following subsection.

## **2.2 Hypotheses on determinants of the industry-level firm size distribution**

Most recent models of the firm size distribution incorporate some type of economic mechanism to generate scale dependence, which is inversely related to the number of large firms in the economy or industry (Rossi-Hansberg et al, 2007). Conceptually, it is meaningful to distinguish between models that generate scale dependence via selection mechanisms (Jovanovic, 1982; Hopenhayn, 1992; Ericson and Pakes, 1995; Klette and Kortum, 2004; Cao and Acemoglu, 2011), models that are centered around frictions in financial markets (Cooley and Quadrini, 2001; Cabral and Mata, 2003; Angelini and Generale, 2008) and endogenous models that generate scale dependence via the efficient allocation of factors of production (Rossi-Hansberg et al, 2007).

In the selection models, scale dependence can be generated by introducing negative productivity shocks that cause more unsuccessful firms to exit the market, as in Luttmer (2007), Hopenhayn (1992) and Ericson and Pakes (1995).

Alternatively, as in Jovanovic (1982), scale dependence can be generated by allowing firms to learn about their own productivity levels once they enter the market.

As suggested in Klette and Kortum (2004), selection can also come about if firms have to change their product lines when competing firms invest in research and development. Klette and Kortum (2004) considers two opposing effects when it comes to the relationship between R&D and firms size. On the one hand, larger firms are endowed with more innovative capital; on the other hand, firms experience diminishing returns to spend more on R&D. Klette and Kortum (2004) show that the opposing effects cancel out if R&D intensity scales with firm size, thus making R&D expenditures relative to revenue independent of the size distribution.

In a survey over the empirical literature between firm size and R&D expenditures, Cohen and Klepper (1996) conclude that there is little evidence that R&D expenditures would increase proportionately over the complete size distribution of firms. Thus, based on Klette and Kortum (2004) an increase in R&D expenditures at the industry level is expected to yield no effect on the industry-level firm size distribution.

Scale dependence can also affect the firm size distribution via constraints or frictions in financial markets. Cabral and Mata (2003) argue that financially constrained small and young firms cannot reach their optimal size in the early stages of their life cycle, which creates a right skewed firm size distribution with fewer large firms.

Angelini and Generale (2008), using individual firm level surveys to ask firms about their financial circumstances, finds financially constrained firm to be smaller with a more right skewed firm size distribution, but still conclude that financial frictions are not likely to be the most important determinant of the shape of the firm size distribution. Nevertheless, an increase in financial frictions related to an industry is expected to have thinning effect on the industry-level firm size distribution.

A different theory is presented by Rossi-Hansberg et al (2007), who assume that scale dependence is generated from the efficient allocation of factors of production. The model is based on the idea that scale dependence, and therefore also the firm size distribution, differs between industries and is governed by the intensity of industry-specific human capital. The smaller the stock of human capital used in an industry, the faster firms in the industry will experience diminishing return to scale, and thus also scale dependence. A higher degree

of dependence, in turn, gets translated into an industry firm size distribution with thinner tails. Their model hence predicts that, conditional on survival, the frequency of large firms in an industry is positively (negatively) related to the intensity of industry-specific human (physical) capital.<sup>3</sup>

In testing their model, Rossi-Hansberg et al (2007) find supportive evidence that the firm size distribution in manufacturing has thinner tails than the more human-capital intensive industry of educational services in the U.S.<sup>4</sup> Thus an increase in industry physical capital is expected to have a thinning effect on the industry-level firm size distribution. Interestingly, Rossi-Hansberg et al (2007) also remark that industries facing greater financial constraints are generally characterized by having a larger share of human capital, which would predict a thinning (and hence, opposite) effect on the firm size distribution.

In addition to the aforementioned determinants, I consider some additional variables that have also been related to the shape of the firm size distribution, either implicitly or explicitly, and are sometimes included in the analysis of firm growth and Gibrat's law (Reichstein et al, 2010; Daunfeldt and Elert, 2010; Delmar and Wennberg, 2010).

First, industry instability, which is defined by Hymer and Pashigian (1962) as the sum of the absolute changes in market shares. A high index value indicates instability in this regard. Kato and Honjo (2006) find that industries with a high degree of concentration (and supposedly thicker tails in the firm size distribution) tend to have more stable market shares. An increase in instability would thus result in less industry concentration as it becomes easier for small firms to survive and compete for resources over time (Delmar and Wennberg, 2010, p.129.) An increase in industry instability, therefore, is expected to have a thinning effect on the industry-level firm size distribution.

Second, firm age. As stated in previous section, several studies find evidence that Gibrat's law can not be rejected for firms with sizes above the industry minimum efficient scale (MES). In industries characterized by mainly old firms, therefore, more firms have already acquired the industry MES, which, conditional on survival, decreases the degree of scale dependence, making the tails

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<sup>3</sup>Although returns to scale are decreasing at the establishment level, they are constant at the industry level. An increase in the share of human capital at the industry level, therefore, leads to a decrease in the share of physical capital. In this model, Zipf's law only emerges in the limit at which either human capital is completely absent from production or is exogenously determined.

<sup>4</sup>A potential caveat regarding the Rossi-Hansberg model is that the concept of human capital specificity lacks empirical support at the industry level. Kambourov and Manovskii (2009) emphasize that specificity is more likely to be found at the occupational level.

of the firm size distribution thicker. Thus industry age is expected to have a thickening effect on the industry-level firm size distribution (Daunfeldt and Elert, 2010).

And third, industry uncertainty. Gabaix (2011) introduce a model where shocks to the largest firms in the economy are able to make a sizable impact on macroeconomic aggregates such as the GDP. Moreover, he shows that the presence of thick tails in the firm size distribution is intimately connected to firm growth volatility. In Daunfeldt and Elert (2010) volatility in firm growth is related to industry uncertainty. In industries with more uncertainty, risk-averse entrepreneurs may be deterred to make entry decisions, which potentially leads to fewer young firms in the industry, and thus less scale dependence. However, Elston and Audretsch (2011) find no evidence that risk attitudes have an effect on the entrepreneurial decision to enter U.S. high-tech industries. Nevertheless, more industry uncertainty is expected to have a thickening effect on the industry-level firm size distribution.

Granted, other variables than those suggested above may be considered to affect the shape of the industry-level firm size distribution. For example, Di Giovanni et al (2011) show that the firm size distribution for exporting firms has a thicker tail than it does for non-exporting firms. However, because trade data are unavailable, exports are excluded from the empirical analysis in this paper. Furthermore, there is also evidence that institutional settings matter in shaping firm size distributions. For instance, Desai et al (2003) show that the size distribution is less right skewed in countries characterized by good protection of property rights and low corruption, where access to capital markets is less constrained by the legal and political climate. In surveying 185 manufacturing firms in Côte d’Ivoire, Sleuwaegen and Goedhuys (2002) observe a firm size distribution characterized by having a “missing middle”, where intermediate sized firms are crowded out by larger firms due to reputation and legitimation factors. However, because I do not have access to cross-country data, institutional variables are also excluded from the empirical analysis.

To summarize, in the remainder of this paper I will test hypotheses regarding the effect of a number of economic variables on the shape of the firm size distribution. These variables are: capital intensity, financial frictions, industry instability, R&D expenditures, industry uncertainty, and industry age.

In accordance with the theoretical predictions, capital intensity, financial frictions and industry instability are hypothesized to have a thinning effect on the industry-level firm size distribution, hence a positive effect on the parameter

$\zeta$  in equation (1). A thickening effect is hypothesized for industry age and industry uncertainty, hence a negatively affect on  $\zeta$ . As for industry R&D the hypothesized effect is ambiguous.

### **3 Data description and empirical strategy**

As mentioned, this paper will use a two-stage approach to determine what affects the shape of the firm size distribution. In the first stage, the parameters of the industry employment firm size distribution are estimated. In the second stage the estimated parameters are used to construct two dependent variables, which are regressed onto a number of explanatory variables. In this section, data and descriptive statistics are presented along with the empirical strategy used to test the hypotheses. Subsection 3.1 presents the data that will be used in the first and second stage of this paper, while subsection 3.2 explains the empirical strategy in more detail.

#### **3.1 Data description**

The data used in this paper covers information on Swedish incorporated firms for the period 1997-2004. The panel consists of accounting information collected from the Swedish Patent and registration office (Patent och registrerings verket, PRV) and prepared by PAR-AB, a Swedish consulting agency that provides detailed market information, frequently used in the Swedish business world. In Sweden, auditing is mandatory for incorporated firms, who are obliged to provide information to PRV. Focusing on incorporated firms rather than, say, proprietorships therefore ensures full access to reliable accounting information (Bradley et al, 2011).

The following actions were taken to improve the quality of the data set: (i) For the sake of statistical accuracy, the years 1995-1996 were dropped from the panel because the data provided only partial coverage of those years. In addition, the data from 2005 were omitted because the information on firms with broken fiscal years had not yet been reported when the data were collected. (ii) To ensure that the data analysis was as consistent as possible with the theoretical models, which is conditional on firm survival, firms that entered, went bankrupt or that exited the market for some other reason during the period under study were excluded. Hence, the analysis is conducted with surviving firms over the period 1997-2004 and thus on the conditional firm size distribution. (iii) Firms with zero employees, which by definition have a size of zero,

were also discarded from the population. (iv) To study the variables affecting the firm size distribution, the analysis is conducted at the 3-digit industry-level distribution, which is the unit of analysis. Firms with missing or invalid NACE-codes were therefore excluded. The 3-digit level is chosen to avoid the problem of the industry scaling puzzle, as firm size distributions are known to break down at the finer 4-and 5-digit levels (Quandt, 1966; Axtell et al, 2006). (v) Because many industries comprise only a handful firms, a minimum number of firms is required for reliable measures. Hence 3-digit industries with fewer than 100 firms were not included.

To study the firm size distribution a measure of individual firm size is required. Measures such as the number of employees, sales, total assets and value added are all commonly used measures of firm size in the economic literature. However, this paper uses each firm’s number of employees as the size metric to remain consistent with the previous literature on the firm size distribution and facilitate comparisons (Rossi-Hansberg et al, 2007; Angelini and Generale, 2008; Cabral and Mata, 2003). Granted, employment is a far from perfect measure, and does not capture all of the complexities of a firm’s size. For instance, it does not necessarily reflect the firm’s total labor force. In business services, for example, many firms hire a portion of their personnel from a human resource consulting firm, and these individuals are not included on their payroll.

The descriptive statistics for employment are presented in Table 1, with a total of 834,599 *NT*-observations distributed across 110 industries in both manufacturing and services. We see that the mean of the variable increases slightly over time. The increasing standard deviations are likely a reflection of the dot-com bubble in 1997-2000.

Based on the employment variable, parameters of the industry employment firm size distribution are estimated in the first stage. They are then regressed onto a number of explanatory variables in the second stage. These variables are summarized in Table 2, while descriptive statistics are presented in Table 3 and a correlation matrix in Table A.2.1 found in Appendix A.2. Because some of the variables include a one-year time lag for the year 1998, the analysis is conducted on the industry-level size distribution using the period from 1999 to 2004. The choices of measurement are discussed further below.

To capture effects of R&D expenditures, industry average for R&D expenditures relative to revenue is included in the analysis, as modeled in Klette and Kortum (2004). Noteworthy is the low number of observations on this variable, which reflects insufficient data from a number of industries.

**Table 1.** Descriptive statistics of employment

Period	The number of Employees				
	Obs.	Mean	Std. Dev.	Min	Max
All years	834,599	11.150	111.046	1	23,321
1999	119,956	10.420	99.216	1	12,836
2000	120,175	11.036	107.088	1	13,452
2001	119,678	11.333	107.659	1	13,029
2002	119,118	11.654	123.986	1	23,321
2003	118,827	11.786	121.226	1	21,842
2004	118,454	11.869	119.978	1	21,023

Note: Employment refers to the number of employees.

According to the theory of Rossi-Hansberg et al (2007), the degree of industry specific human capital should have a thickening effect on the firm size distribution. However, the PAR-data do not contain information on human capital. Nevertheless, under the assumption of constant returns to scale at the industry level, an increase in human capital implies a decrease in the share of physical capital (Rossi-Hansberg et al, 2007). A variable for industry-level physical capital is therefore included to capture the effect of human capital. Physical capital is calculated by taking the industry average of tangible fixed assets as a share of revenues. This proxy differs from the one used in Rossi-Hansberg et al (2007), in which an industry’s physical capital share is calculated as one minus labor’s share of value added. Because the capital intensity measure contains a number of outliers in the upper tail, a truncation at the 95th percentile is used at the firm level.<sup>5</sup>

There is an ongoing debate in the financial literature regarding how to measure financial frictions. In Daunfeldt and Elert (2010), financial frictions are approximated using the industry average for liquidity relative to revenue. Alternatively, financial constraints can be measured using cash flow (Fazzari et al, 1988; Blundell et al, 1992), even though the appropriateness of this measure has been questioned by Kaplan and Zingales (2000). Here, financial constraints are measured as in Daunfeldt and Elert (2010), using industry average for liquidity relative to revenue.

<sup>5</sup>The physical capital share as defined by Rossi-Hansberg et al (2007) was constructed but found unreliable due to available information on value added, which were shaky and included a significant number of missing values. Alternatively, one can also measure physical capital’s share of production by estimating an underlying production function, but this method has not been attempted here.

**Table 2.** Definition of the explanatory variables

Variable (Hypothesis)	Variable description	Calculation <sup>a</sup>
Industry R&D (Ambiguous)	The industry average of R&D relative to revenue.	$\frac{1}{n} \sum_{i=1}^n \frac{R\&D_{i,j,t}}{revenue_{i,j,t}}$
Capital intensity (Thinning)	The industry average of physical capital relative to revenue.	$\frac{1}{n} \sum_{i=1}^n \frac{tangible\ fixed\ assets_{i,j,t}}{revenue_{i,j,t}}$
Financial friction <sup>c</sup> (Thinning)	The industry average of liquidity relative to revenue.	$\frac{1}{n} \sum_{i=1}^n \frac{liquidity_{i,j,t}}{revenue_{i,j,t}}$
Industry instability (Thinning)	Sum of the absolute changes in industry market shares (Hymer and Pashigian, 1962). <sup>b</sup>	$\sum_{i=1}^n \left  \Delta \frac{revenue_{i,j,t}}{\sum_{i=1}^n revenue_{i,j,t}} \right $
Industry uncertainty (Thickening)	The standard deviation of firm growth in the industry.	St.dev <sub><i>j,t</i></sub> $\left( \ln \left( \frac{revenue_{i,j,t}}{revenue_{i,j,t-1}} \right) \right)$
Industry age (Thickening)	The industry average of firm age.	$\frac{1}{n} \sum_{i=1}^n age_{i,j,t}$
Industry age <sup>2</sup> (Ambiguous)	The industry average of firm log-age squared	$\left( \frac{1}{n} \sum_{i=1}^n \left( age_{i,j,t} - \frac{1}{n} \sum_{i=1}^n age_{i,j,t} \right)^2 \right)$
Industry growth (Control)	The change in sum of log revenues from <i>t</i> - 1 to <i>t</i> in the industry. <sup>b</sup>	$\Delta \sum_{i=1}^n \ln (revenue_{i,j,t})$
Industry size (Control)	The total number of firms within an industry relative to all industries, measured in logarithms	$\frac{\sum_{i=1}^n \ln(i,j,t)}{\sum_{i=1}^N \ln(i,t)}$

<sup>a</sup>Note: firm (*i*), industry (*j*), year (*t*); number of firms in industry *j* (*n*).

<sup>b</sup>The symbol  $\Delta$  refers to the time-difference operator.

<sup>c</sup>Observe that more financial friction is associated with a smaller value, which means that liquidity in terms of revenue is less.

**Table 3.** Descriptive statistics for the explanatory variables

Indep.variables	Obs.	Mean	Std. Dev.	Min	Max
Industry R&D	405	7.962	1.538	1.386	11.495
Capital intensity	770	0.229	0.333	0.025	3.382
Financial friction	770	0.305	0.229	0.057	1.597
Industry instability	660	0.180	0.09	0.003	1.012
Industry uncertainty	660	0.162	0.036	0.06	0.315
Industry age	770	14.525	3.063	5.124	23.782
Industry growth	660	29.198	57.623	-25.94	502.231
Industry size	770	0.009	0.014	0.008	0.092

The industry instability index was calculated as suggested in Kato and Honjo (2006), hence as the sum of the absolute changes in market shares, but where market shares are measured in terms of revenue. Moreover, industry uncertainty, that is micro-level volatility, was calculated by the cross-sectional standard deviation of (log) firm growth in respective industry, measured in terms of revenue (Daunfeldt and Elert, 2010). Furthermore, industry age was calculated by taking the industry average over firm age. A squared term was also calculated to capture any possible non-linear age effects. In the squared age-term the industry mean was subtracted from each firm to reduce multicollinearity with the age variable.

To correct for differences in industry size and growth, two controls are also included. Industry size is here defined as the total number of firms in the industry calculated as a share of the firms in all industries. However, because I study surviving firms over the period the variable is subsumed in the time-fixed effects, and not presented in the results.

As for industry growth, it was calculated based on the change in total industry (log) revenues rather than a change in numbers of employees to avoid possible problems of endogeneity. All explanatory variables have been lagged one period to avoid simultaneity issues, which is why, once again, the empirical analysis of the firm size distribution is conducted over the period 1999-2004.

### 3.2 Empirical strategy

To model the firm size distributions, the relationship between each firm's relative rank in the firm size distribution and its size is used. The firms are first sorted in descending order with respect to their current size according to  $size_{1,j,t} > size_{2,j,t} > \dots > size_{n,j,t}$ . Next, in each time period  $t$  and industry  $j$ , the largest

firm,  $size_{1,j,t}$ , is assigned a rank of one; the second largest,  $size_{2,j,t}$ , is assigned a rank of two; and the smallest firm is assigned the same rank as the number of  $n$  firms that are active in the industry.

The rank-size relationship is a simple tool that is commonly used to determine whether the firm size distribution follows a power law. If so, there should be a log-linear relationship between a firm's rank and its size, given approximately by

$$rank \simeq n (size/size_{min})^{-\zeta}, \quad (2)$$

where  $\zeta \in (0, \infty]$  is the power law exponent defined in (1).<sup>6</sup> The term  $size_{min}$  is a size boundary and refers to the smallest size above which the relationship in (2) is likely to hold. Although the characterization in (2) is approximate, it is motivated by  $i/n = E \left[ (size_{i,t}/size_{min,t})^{-\zeta} \right]$  for  $i = 1, \dots, n$ , assuming that (1) is the correct description of the firm size distribution (Gabaix, 2009).

Following Ioannides et al (2008), Rosen and Resnick (1980) and Soo (2005), the determinants of the firm size distribution are studied using a two-stage model. In the first stage, the power law exponent  $\zeta_{j,t}$ , along with the possible quadratic deviations  $\gamma_{j,t}$ , is estimated for each industry at the 3-digit level. Linearizing (2) makes it possible to estimate the following regression:

$$\begin{aligned} \ln(rank_{i,j,t} - 1/2) &= \alpha_{j,t} - \zeta_{j,t} \ln(size_{i,j,t}) + \gamma_{j,t} \ln^2(size_{i,j,t} - s^*) + \epsilon_{i,j,t}, \\ w.r.t. \quad size_{i,j,t} &\geq size_{min}, \end{aligned} \quad (3)$$

where  $rank_{i,j,t}$  is the relative rank of firm  $i = 1, \dots, n$  in industry  $j = 1, \dots, 110$  during year  $t = 1999, \dots, 2004$ . Gabaix and Ibragimov (2011a) shows that subtracting  $1/2$  from the rank effectively reduces small sample bias.

If the distribution is a power law distribution, the regression should produce a straight line with the slope  $-\zeta_{j,t}$ . To test the significance of the slope coefficient, a quadratic term  $\ln^2(size_{i,j,t})$  is added. However, to ensure that the estimate of  $\hat{\zeta}_{j,t}$  is not affected, Gabaix and Ibragimov (2008) use a shift

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<sup>6</sup>When plotting (2) it is customary to transform the abscissa and ordinate into logarithmic scales, i.e., into a log-log plot. One should, however, be cautious when plotting (2) since other probability distributions may display something very similar to the ubiquitous straight line. Eeckhout (2004; 2009) shows that the log-log plot can distort the data, especially for large sizes that appears to deviate from the power-law benchmark. A slight deviation from the straight line can thus be fully compatible with a power law. Conversely, a good ocular fit for small firms need not signal a power law since the log-log plot shrinks the standard deviation.

parameter  $s^* = \frac{\text{cov}(\ln^2(\text{size}_{i,j,t}), \ln(\text{size}_{i,j,t}))}{2\text{var}(\ln(\text{size}_{i,j,t}))}$  to recenter the quadratic term. If the firm size distribution does not exhibit a power law distribution, the estimate of  $\gamma_{j,t}$  will become significant, and the regression in (3) will produce a concave relationship between a firm's rank and its size in industries with thinner tail.

However, because the ranking procedure creates positive autocorrelation in the error term  $\epsilon_{i,j,t}$ , standard deviations provided by the statistical software program are not reliable. Nevertheless, under the null hypothesis  $\sqrt{2n}/\hat{\zeta}_{j,t}^2$  is asymptotically Gaussian, and the power law hypothesis can be tested using  $|\hat{\gamma}_{j,t}| > 2.57\hat{\zeta}_{j,t}^2/\sqrt{2n}$ . If the inequality holds, the quadratic term is significant at the 99 percent level, and the null hypothesis of a power law can be rejected (Rozenfeld et al, 2011).

In essence, if the null hypothesis of a power law is rejected it means that the firm size distribution can take any other shape. Most likely, a rejection of a power law will be because the firm size distribution has thinner tails, producing a concave downward deviation from the power law benchmark. There is also the possibility that the null hypothesis of a power law is rejected because industries have more large firms than accounted for by the power law. In that case the deviation is convex producing an upward deviation from the power law benchmark. The null hypothesis of Zipf's law is not rejected if  $\zeta_{j,t} = 1$  falls inside a 95 percent confidence interval of  $\hat{\zeta}_{j,t}$ .

Before estimating (3), however, the issue of an appropriate size boundary,  $\text{size}_{i,j,t} \geq \text{size}_{min}$ , needs to be addressed. Typically, (3) is estimated for all firms, including those that are very small. Given that a power law distribution is generally a poor fit for smaller firms, running the regression on all firms is likely to yield biased estimates and lead us to reject the null hypothesis of a power law. If, on the other hand, the regression is run for some arbitrarily chosen boundary that is too high, valuable data are excluded from the analysis, leading to a loss of power (Clauset et al, 2009). To locate the appropriate boundary, a search algorithm is used as suggested by Clauset et al (2009). The method is fairly simple and is based on evaluating (3) for an array of different boundaries. For each potential boundary, with index  $k$ , the empirical distribution is compared to the theoretical power law,  $P(\text{size} > x) = \left(\text{size}_{i,j,t}/\text{size}_{min}^{(k)}\right)^{-\hat{\zeta}_{j,t}}$  using the one-sided Kolmogorov-Smirnov goodness of fit test. Whichever threshold produces the smallest statistic is then used (see Appendix A.1 for a detailed description). The appropriate boundary is then estimated for each industry and time period.

One important difference between this approach and the one suggested in

Clauset et al (2009) is that least squares are used instead of maximum likelihood estimation and the Hill (1975) estimator. This choice is made for three reasons. First, compared to the Hill estimator, OLS is usually more robust to deviations from a power law (Gabaix, 2009). Second, least squares makes it possible to test the null hypothesis of a power law simultaneously as the parameters are estimated. Third, and most important, using least squares makes it possible to characterize the quadratic deviation  $\hat{\gamma}_{j,t}$  when the null hypothesis is rejected, which is not possible with for instance the Hill estimator.

Next, a second-stage regression is estimated using the estimated slope and quadratic coefficients from the first stage as dependent variables in separate regressions. As far as I know, this methodology has not previously been suggested for use in studies of the firm size distribution. Although the aforementioned studies of Ioannides et al (2008), Rosen and Resnick (1980) and Soo (2005) were conducted in the area of urban growth, the same empirical strategy should be applicable to the study of the firm size distribution. The resulting estimates  $\hat{\zeta}_{j,t}(\mathbf{X}_{j,t-1})$  and  $\hat{\gamma}_{j,t}(\mathbf{X}_{j,t-1})$  are hypothesized to be functions of the set of explanatory variables  $\mathbf{X}_{j,t-1}$  described in Table 2.

I proceed to estimate the following model,

$$Y_{j,t} = \mu_j + \delta_t + \mathbf{X}_{j,t-1} \Theta + \xi_{j,t}, \quad (4)$$

where the estimated coefficients  $\hat{\zeta}_{j,t}$ , and  $\hat{\gamma}_{j,t}$  from (3) are used as dependent variables  $Y_{j,t}$ . Two separate regressions are estimated, one with the power law exponent  $\hat{\zeta}_{j,t}$  as the dependent variable and one with the quadratic deviation  $\hat{\gamma}_{j,t}$  as the dependent variable. The first regression, which uses  $\hat{\zeta}_{j,t}$  as the dependent variable, only include industries for which a power law could not be rejected, whereas the second, which uses  $\hat{\gamma}_{j,t}$  as the dependent variable, only include industries for which a power law could be rejected. The latter specification makes it possible to examine potential deviations of the firm size distribution from the power law benchmark and, thus, to explore whether power law decay is caused by some mechanism other than that for distributions with thinner tails. Importantly, when a power law is rejected,  $\hat{\gamma}_{j,t}$  is acquired by running the first-stage regression again without a size threshold (setting  $size_{min} = 1$ ) and thereby including the complete distribution of firms in (3).

Specification (4) is a panel in which the intercept  $\mu_j$  allows for industry-specific fixed effects. Because the firm size distribution is conditional with respect to entry and exit, the control for industry size, measured as the number of

firms within each industry, becomes time invariant and is captured by  $\delta_t$ . The parameter  $\delta_t$  is a dummy variable that captures time-variant heterogeneity and controls for contingent business cycle effects.

In a two-stage setting such as ours, in which the coefficients from the first stage are used as dependent variables in the second stage, OLS is known to provide inefficient results if measurement error occurs in the first stage (Lewis, 2000). To resolve this problem, Lewis (2000) advocates the use of feasible generalized least squares (FGLS). However, FGLS has poor small sample properties because  $T \rightarrow \infty$  is required. The time period studied here covers only 6 years, which makes this estimator unsuitable, as it is likely to underestimate the standard error. Similarly, as Hoechle (2007) notes, the Beck and Katz (1995) estimator with panel corrected standard errors (PCSE) generally performs poorly for large- $N$  small- $T$  samples.

The Driscoll and Kraay (1998) method has been shown to be superior for small- $T$  samples, even if  $N$  grows large, and the estimator is robust to general forms of cross-sectional dependence, heteroskedasticity and an MA(q) autocorrelated error term (Hoechle, 2007). The estimator is also compatible with within-subject effects and makes it possible to control for industry-specific fixed effects.<sup>7</sup>

In total, I use four different strategies to estimate (4) when the slope coefficient  $\hat{\zeta}_{j,t}$  is the dependent variable. First, I use weighted least squares (WLS). The weights are defined by the inverse of the standard deviations of  $\hat{\zeta}_{j,t}$  and are calculated using  $(\hat{\zeta}_{j,t})^{-1} (n/2)^{1/2}$ , which gives more weight to industries with small standard errors in the estimated power law parameter  $\hat{\zeta}_{j,t}$  (Ioannides et al, 2008). Second, because industry fixed effects may affect the results, I use a model with a fixed effect within estimator (FE). In both the WLS and the FE regressions, I use Driscoll and Kraay standard errors. Lastly, because  $Y_{j,t} > 0$ , the model has a limited dependent variable, I therefore estimate a log-linear specification of (4) suggested from a Boxcox transformation, to avoid predicted negative values. Finally, to retain  $E[Y_{j,t}|\mathbf{X}_{j,t}]$  and simultaneously address the limited dependent variable issue, I also use a gamma quasi maximum likelihood (QMLE) estimator, sometimes referred to as GLM gamma regression, with a

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<sup>7</sup>To test for the validity of within transformation, I perform a modified Hausman test as suggested in Hoechle (2007). Woolrige (2010, p.288) shows that the standard Hausman test is not applicable if not  $\mu_j$  and  $\xi_{j,t}$  are i.i.d. advocating a panel correction test. Under the general form of dependence, however, the panel corrected Hausman test is also invalid. To ensure that the test is valid under the minimal assumption, the auxiliary equation in the Hausman test is estimated using Driscoll-Kraay standard errors as suggested by Hoechle (2007).

log-link as suggested by Woolridge (2010 p.740). The log-linear and QMLE results are presented in Appendix A.2. To estimate (4) when the quadratic coefficient  $\hat{\gamma}_{j,t}$  is the dependent variable I resort to OLS, which is further explained in the result section.

## 4 Results

This section covers the empirical results. First, the results of the first stage regressions are presented. In this regression, the parameters of the firm size distribution are estimated based on equation (3). Then, I present the results obtained by regressing the shape parameters onto the explanatory variables as specified in equation (4).

### 4.1 How is the firm size distribution shaped?

The results obtained from the first stage are presented in Table 4. As the number of regressions is too large for all of the results to be presented here, Table 4 shows a summary of the results.<sup>8</sup> For 611 out of the 770 year-industry specific regressions, the null hypothesis of a power law could not be rejected. Within this group of 611 regressions, Zipf’s law could not be rejected in 445 instances for which  $\zeta_{j,t} = 1$  was found inside a 95 percent confidence interval of  $\hat{\zeta}_{j,t}$ . The estimated boundary size was here found to be 28 employees on average.

In the 159 out of the 770 cases for which the null hypothesis of a power law were rejected, the first stage regression were run again without a size boundary. For these year-industry regressions the average deviation was found to be -0.15, which indicates a firm size distribution with thinner tails than the power law. Within this group of 159 instances only 7 regressions showed a convex deviation from the power law benchmark, for which the firm size distribution has a thicker thicker tail than a power law.

The results thus confirm the presence of a power law in the firm size distribution, as this hypothesis could not be rejected in 78.6 percent of the instances considered. In total, the empirical regularity known as Zipf’s law could not be rejected in 57.3 percent of all the 770 year-industry regressions.

To better illustrate the results, Figure 1 displays the estimates for the power

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<sup>8</sup>All of the results can be obtained from the author upon request.

**Table 4.** Results of the first-stage regression for the period 1999-2004

Variable:	Mean	St.dev	Min	Max
<i>No. reg. not rejecting a power law (n=611)</i>				
Slope coefficient <sup>a</sup> : $\hat{\zeta}_{j,t}$	1.304	0.565	0.425	4.936
Size threshold <sup>b</sup>	28.088	42.726	1	339
<i>No. reg. rejecting a power law (n=159)</i>				
Quadratic coeff. <sup>c</sup> : $\hat{\gamma}_{j,t}$	-0.15	0.122	-.656	0.081

<sup>a</sup>Slope coefficients from the first-stage equation in industries for which the power law hypothesis could not be rejected.

<sup>b</sup>'Optimal' size threshold derived using the algorithm described in Appendix A.1.

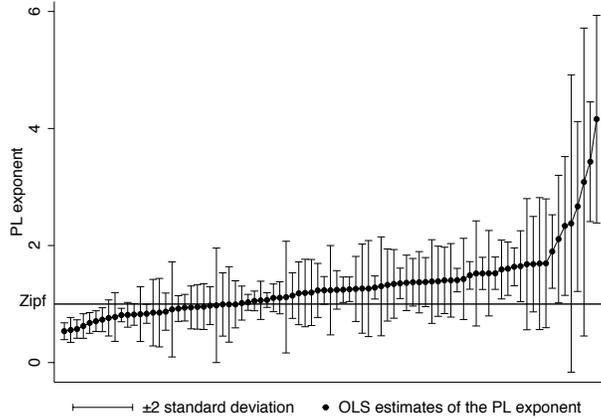
<sup>c</sup>Curvature coefficient from the first-stage equation in industries for which the power law hypothesis could be rejected. The regressions were conducted for all firms without a size threshold.

law exponent  $\hat{\zeta}_{j,t}$  with a 95 percent confidence interval for the examined 110 industries from the year 2004. The horizontal line represents Zipf's law, in which the exponent takes a value of one. As can be seen from the figure, even if most of the point estimates deviate from the law, it is rarely rejected at a 5 percent significance level.<sup>9</sup>

To examine the non-parametric properties of the results, the kernel density estimates for the power law exponent and the quadratic term are shown in Figure 2, both with a Gaussian overlay. Whereas the density of the quadratic coefficient in Figure 2 (b) looks roughly Gaussian, the density of the exponent in Figure 2 (a) exhibits a more lognormal shape. A Jarque-Bera test, however, rejects normality in both cases. Although it cannot be seen clearly in Figure 1 there is some clustering around Zipf's law. This is more visible in Figure 2 (a), in which Zipf's law roughly represents the mode of the kernel density. Although a number of industries display significantly thinner tails than is typical for a power law distribution, the overall results are consistent with previous empirical findings regarding a power law in the firm size distribution, and also roughly consistent with Zipf's law (Axtell, 2001; Okuyama et al, 1999; Fujiwara et al, 2004).

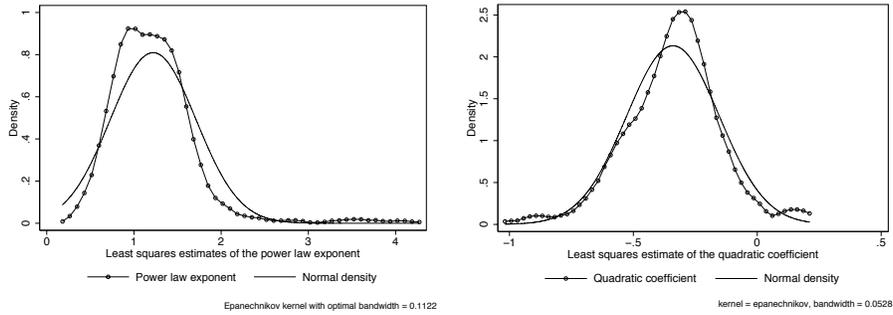
The next two subsections present the results of the second-stage procedure, in which the parameter estimates from the first stage are regressed onto a number of explanatory variables.

<sup>9</sup>This finding may reflect the limitations of regression analysis, and it could be helpful to use a more powerful test to distinguish the power law distribution from other distributions – for instance, the test advocated in Malevergne et al (2011).



Note: Figure 1 shows the first-stage results for the year 2004 with 95 percent confidence intervals computed using the asymptotic standard error  $\pm 2\hat{\zeta}_{j,t}(n/2)^{-1/2}$  (Gabaix, 2009). A horizontal line is added for a power law exponent of 1, which represents Zipf's law (PL on the vertical axis refers to power law).

**Figure 1.** Estimates of the slope coefficient obtained from the first-stage regression with 95 percent confidence intervals for the year 2004.



(a) Density plot of the slope coefficient ( $\hat{\zeta}_{j,t}$ ) (b) Density plot of the quadratic coefficient ( $\hat{\gamma}_{j,t}$ )

Note: Figure 2 (a) shows the kernel density plot for the least squares estimates of the power law exponent when the power law hypothesis cannot be rejected. Figure 2 (b) shows the same plot for the estimated quadratic term when the power law hypothesis can be rejected. The kernel densities are computed using the Epanechnikov function at 50 points with an optimal bandwidth that minimizes the mean integrated squared error assuming that the data is normal distributed. In both figures, the fitted normal densities have been overlaid.

**Figure 2.** Density plots of the coefficients obtained from the first-stage regression for the period 1999-2004

## 4.2 What determines the shape of the firm size distribution?

Table 5 shows the results for various specifications obtained when the power law exponent  $\hat{\zeta}_{j,t}$  is used as the dependent variable. Note that the positive effect of  $\mathbf{X}_{j,t}$  should be interpreted as a thinning effect on the firm size distribution, and a negative effect should be interpreted as a thickening effect.

In the WLS regressions (1-3), the degree of capital intensity is found to have a positive effect, which validates the prediction by Rossi-Hansberg et al (2007) that a higher level of physical capital (human capital) is correlated with firm size distributions with thinner (thicker) tails. The effect remains positive and significant when industry fixed effects have been controlled for (regressions 5-7), although the effect increases somewhat.

Here, a higher value for the financial constraint measure should be interpreted as indicating that fewer frictions are present in the industry. According to the hypothesis being considered, fewer frictions should have a thickening effect on the firm size distribution and generate a negative effect on the power law exponent. In both the WLS and the FE regressions (1-7), the effect is negative and significant. Conversely, greater frictions should generate a firm size distribution with thinner tails because, according to Cabral and Mata (2003), smaller firms are restricted from reaching their desired sizes (potentially because they cannot access the financial capital market).

The results for capital intensity and liquidity partially resolve the puzzle identified by Rossi-Hansberg et al (2007): that financial frictions are usually more pronounced in industries with little physical capital. The positive pairwise correlation (0.121\*\*) between the two variables, as indicated in Table A.2.1, seems to suggest that financial frictions are more pronounced in industries with little physical capital and that capital intensity will have the opposite sign (Rossi-Hansberg et al, 2007). The results, however, indicate the existence of an interaction effect between the two variables, which might favor either theory.

Although industry uncertainty has no discernible effect, the index of instability is found to be positive and significant under WLS. However, once the industry fixed effects have been corrected for, the sign of the effect changes, and it becomes negative, although it is now only significant at the 95 percent level when the full set of variables is included in regression (7).

One interesting finding is the role played by industry R&D. The expected effect from this variable was ambiguous, however the observed negative effect would suggest that industry R&D has a positive effect on the number of large

**Table 5.** Results from the second-stage regression for the period 1999-2004

Dependent variable: $\hat{\zeta}_{j,t}$	WLS		WLS		WLS		FE		FE	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Capital intensity $_{t-1}$	0.154*** (0.037)	0.156*** (0.039)	0.213*** (0.054)	-0.146*** (0.028)	1.05*** (0.329)	1.239*** (0.325)	2.727*** (0.808)			
Financial friction $_{t-1}$		-0.444*** (0.013)	-0.481*** (0.034)	-0.293*** (0.076)		-1.3** (0.624)	-0.664** (0.327)			
Instability index $_{t-1}$		0.515** (0.201)	0.404*** (0.206)	0.339*** (0.102)		-0.163 (0.164)	-0.698** (0.315)			
Industry uncertainty $_{t-1}$		0.489* (0.28)	0.119 (0.312)	-0.193 (0.923)		0.336 (0.312)	-0.597 (0.584)			
Industry age $_{t-1}$			-0.007 (0.012)	-0.045*** (0.014)			0.673*** (0.091)			
Squared industry age $_{t-1}$			-0.025 (0.024)	0.079*** (0.027)			-0.053 (0.134)			
Industry growth $_{t-1}$			0.001*** (0.000)	0.001*** (0.000)			-0.000 (0.000)			
Industry R&D $_{t-1}$				-0.036*** (0.011)			-0.017** (0.007)			
Constant	1.09*** (0.006)	1.037*** (0.038)	1.354*** (0.142)	1.87*** (0.178)	1.028*** (0.067)	1.548*** (0.255)	(omitted)			
Disc./Kray s.e.	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Time fixed effects	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Hausman (Disc./Kray)			[0.000]		[0.000]	[0.000]	[0.000]			[0.000]
Obs	519	429	429	218	519	429	218			
Within R <sup>2</sup>	0.121	0.168	0.313	0.50	0.01	0.04	0.127			

Note: Standard errors are listed in brackets. The significance level (\*) is 90 percent, (\*\*) is 95 percent and (\*\*\*) is 99 percent. In the weighted least squares (WLS) analysis, I use analytical weights defined as the inverse of the standard error of the power law exponent from the first-stage regression. These weights are computed using  $(\hat{\zeta}_{j,t})^{-1} (n/2)^{1/2}$  and give more weight to industries with small standard errors in the estimated power law exponent  $\hat{\zeta}_{j,t}$  (Ioannides et al., 2008). All regression are run with Driscoll and Kray standard error that is robust to very general forms of inter temporal and cross-sectional dependence. The fixed effect estimator is computed by within transformation. All variables have been lagged one time period to escape potential problem with simultaneity. The dependent variable refers to the slope coefficient from the first-stage equation (3) for year-industries where a power law could not be rejected.

firms in the industry. This results may indicate that the larger endowment of innovative capital for large firms outweighs the diminishing return to invest in more R&D. When industry R&D is included in regression (4), the sign of capital intensity changes from positive to negative. However, when fixed effects are corrected for, the sign of capital intensity remains positive, which suggests that the negative sign in regression (4) is driven by industry fixed effects.

To test the robustness of the model, Table A.2.2 in Appendix A.2 presents the results when the limited response variable has been taken into consideration. These results generally confirm the findings in Table 5, except when industry R&D is included in the analysis. This suggests the presence of some spurious correlation between the capital variable and the R&D variable, despite their insignificant correlation in Table A.2.1. Another possible explanation could be the fewer number of observations for expenditures in R&D, which likely makes the empirical model less robust.

### 4.3 Are the determinants unique to industries characterized by a power law?

By regressing the quadratic coefficient  $\hat{\gamma}_{j,t}$  on the same set of variables as for the power law exponent, one obtains a more general understanding of what shapes the size distribution for industries with thinner tails than a power law distribution. Importantly, because the hypothesis of a power law distribution could be rejected at the optimal size threshold, the quadratic deviation is estimated when all firms are included during the first stage.

The number of observations here is substantially smaller because the power law hypothesis in the first stage were rejected in only 159 cases, which generates a strongly unbalanced panel. Therefore, the fixed effect estimator is likely to be biased, and Table 6 only shows the results of four specifications of the OLS regression.

The regressions were estimated using Driscoll-Kraay standard errors, but the weights were omitted.<sup>10</sup> To facilitate the interpretation of the results, two modifications to  $\hat{\gamma}_{j,t}$  were made. First, because 7 of the observed deviations from the power law distribution were convex, these deviations were excluded because the focus here is on deviations in which the tail was thinner than in the power law null hypothesis. Second, to simplify the interpretation of the effect of the

<sup>10</sup>It should be possible to use weights similar to  $\hat{\gamma}_{j,t} - 2.57\hat{\zeta}_{j,t}^2/\sqrt{2n}$  that more strongly emphasize industries for which the significance of  $\hat{\gamma}_{j,t}$  is stronger. However, this method was not used here.

explanatory variables, the absolute value of the quadratic coefficient was taken, which yielded a dependent variable of the form  $|\hat{\gamma}_{j,t} < 0|$ .

The results are presented in Table 6, where a positive effect should be interpreted as having a thinning effect on the distribution. Here, however, this effect should be interpreted as the result of more pronounced concavity. For both capital intensity and liquidity, the results when using  $\hat{\gamma}_{j,t}$  as the dependent variable persist, where higher capital intensity and financial frictions have a thinning effect on the industry-level size distribution. As in Table 5, adding industry R&D to the regression alters the effect of capital intensity, but instead of making the sign negative, this addition makes the influence of capital intensity insignificant. In regression (2), the instability index has a positive effect, as was the case for WLS in Table 5, which suggests that more unstable market shares tend to generate a thinner tail in the firm size distribution, leading us initially to accept the hypothesis. This effect, however, is not present in the full specification in regression (4).

The similar results found for  $\hat{\gamma}_{j,t}$  and  $\hat{\zeta}_{j,t}$  regarding capital intensity and financial friction suggest that the same causal mechanisms that determine the power law exponent for larger firms also determine the concave deviation from a power law when firms of all sizes are taken into consideration. Thus, based on these results power laws in the firm size distribution may not be imbued with any unique character. However, the significant negative effects from industry uncertainty in regressions (2-3) (Table 6), would indicate that more uncertainty has a thickening effect on the firm size distribution, but only in industries where a power law could be rejected.

**Table 6.** Results from the second stage regression for the period 1999-2004

Dependent variable: $ \hat{\gamma}_{j,t} < 0 $	OLS	OLS	OLS	OLS
	(1)	(2)	(3)	(4)
Capital intensity $_{t-1}$	0.147*	0.019***	0.234***	0.049
	(0.078)	(0.003)	(0.053)	(0.034)
Financial friction $_{t-1}$		-0.076***	-0.042**	-0.110***
		(0.017)	(0.02)	(0.008)
Instability index $_{t-1}$		0.252**	-0.013	0.128
		(0.102)	(0.112)	(0.08)
Industry uncertainty $_{t-1}$		-0.993***	-1.144***	-0.149
		(0.143)	(0.108)	(0.202)
Industry age $_{t-1}$			-0.101**	-0.012***
			(0.004)	(0.002)
Squared industry age $_{t-1}$			-0.021	0.044***
			(0.021)	(0.008)
Industry growth $_{t-1}$			-0.000	0.000***
			(0.000)	(0.000)
Industry R&D $_{t-1}$				-0.017*
				(0.004)
Constant	0.133***	0.273***	(omitted)	0.315***
	(0.014)	(0.022)	-	(0.056)
Disc./Kray s.e.	yes	yes	yes	yes
Time fixed effects	yes	yes	yes	yes
Obs	129	111	111	72
Within R <sup>2</sup>	0.117	0.233	0.295	0.329

Note: Standard errors are listed in brackets. The significance level (\*) is 90 percent, (\*\*) is 95 percent and (\*\*\*) is 99 percent. All regression are run using Driscoll and Kray standard errors that is robust to very general forms of inter temporal and cross-sectional dependence. Given the small sample and the highly unbalanced panel, the fixed effect estimates are not displayed. All regressions are run without size thresholds for industries for which the power law hypothesis could be rejected at the optimal threshold. Because of extreme outliers in the data for the capital variable for NACE 672, this industry has been deleted from the sample. The dependent variable refers to the quadratic coefficient from estimating the first-stage regression in equation (3), for year-industries where a power law could be rejected.

## 5 Concluding remarks

This study addressed the lack of empirical research in the industrial organization literature on the firm size distribution. A two-stage empirical model was constructed in which the parameters of the industry-level firm size distribution were estimated during the first stage. Drawing on recent theoretical insights, the cross-industry variation in the firm size distribution was then modeled in a regression framework with a number of explanatory variables in the second stage. Two dependent variables were considered: the exponent that characterizes the power law distribution and the quadrature that captured any concave deviations from the power law benchmark. The dataset used was composed of surviving incorporated Swedish firms that were sorted into groups according to their 3-digit industry in the period 1997-2004, for which the empirical analysis considered the period 1999-2004.

The results seem to confirm the presence of a power law in the firm size distribution, as this hypothesis could not be rejected in 78.6 percent of the instances considered. In addition, the empirical regularity known as Zipf's law could not be rejected in 57.3 percent of all cases. In the second stage regression analysis, capital intensity, financial frictions, and arguably industry instability were found to have a significant and thinning effect on the firm size distribution. The similar results on capital intensity and financial friction, regardless the dependent variable considered, suggest that industries with firm size distributions described by a power law are not endowed with any special mechanism that differentiates them from industries in which the firm size distribution has thinner tails.

The results of this study suggest that lower levels of financial friction have a significant positive effect on the proportion of large firms in an industry. To encourage the growth and production of small firms, governmental policy may hence seek to decrease financial frictions by making financial capital more readily available. For instance, Guner et al (2008) argue that government regulations that target firms of different sizes account for a significant portion of the observed differences between countries. The researchers argue that size-dependent policies intended to either restrict the production of large firms or encourage that of small firms have a substantial effect on the output, productivity and distribution of firms.

However, consistent with the predictions of this study, I also find that the degree of physical capital has a moderating effect on the proportion of large firms,

although financial frictions should in theory be more prevalent in less capital intensive industries. Policymakers should therefore take caution in developing related economic policies until this issue has been satisfactorily resolved.

Future studies could benefit from investigating the firm size distribution from a dynamic perspective. For example, by including a lagged dependent variable in the second-stage regression, one could study the long-term properties of the size distribution as the power law exponent reaches a steady state. In this setting, it would be possible to investigate the effect of various exogenous shocks over time.

Moreover, the method outlined in this paper is nonrestrictive, and other variables could be included that might affect the shape of the firm size distribution. For instance, instead of cross-industry variations, regional cross-variation could be used to study the effect of agglomeration economies. Finally, the detection of power laws in empirical phenomena will become more interesting if the size threshold issue is taken seriously, and researchers might fruitfully pursue a Markov chain analysis of the transmission mechanisms for potential *Pareto firms* that enter the 'power-law line'.

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## A Appendix

### A.1 Recipe for locating the appropriate size boundary

To extract the appropriate size boundary, a search algorithm is used, as advocated by Clauset et al (2009). The boundary is found by minimizing the Kolmogorov-Smirnov statistic for the empirical distribution and the counter cumulative distribution in (1) for an array of firm sizes as follows:

1. For each boundary, denoted by  $size_{min}^{(k)}$ , estimate equation (3) for firms with a lower rank than  $size_{min}^{(k)}$  such that  $size_{1,j,t} > size_{2,j,t} > \dots > size_{min}^{(k)}$ . Following Clauset et al (2009) use the array of unique values of  $size_{i,j,t}$ , within each industry and year, as candidates for the boundary  $\hat{size}_{min}$ .
2. Compute the one-sided Kolmogorov-Smirnov (KS) statistic for the empirical distribution and the estimated power law. For each  $size_{min}^{(k)}$  the KS test using,

$$KS_{j,t}(k) = \max_{size_{i,j,t} > size_{min}^{(k)}} \left| \frac{1}{n} \sum_{i=1}^n I_{size_{i,j,t} \geq size_{min}^{(k)}} - \left( size_{i,j,t} / size_{min}^{(k)} \right)^{-\hat{\zeta}_{j,t}} \right|,$$

where the left expression within brackets is the empirical cumulative distribution function and  $I$  the indicator function taking value of 1 if  $size_{i,j,t} \geq size_{min}^{(k)}$  and a value of zero otherwise. The coefficient  $\hat{\zeta}_{j,t}$  refers to the least squares estimate from the regression in (3) evaluated above the size threshold  $size_{min}^{(k)}$ .

3. Whichever  $size_{min}^{(k)}$  that minimizes KS is taken as the probabilistically appropriate size threshold  $\hat{size}_{min}$ .

An important feature of the approach outlined here is that least squares are used instead of maximum likelihood and the Hill (1975) estimator as in Clauset et al (2009).

## A.2 Tables

**Table A.2.2.** Results obtained using alternative regression specifications for the period 1999-2004

Dependent variable:	WLS	FE	Gamma QMLE
	$\log(\hat{\zeta}_{j,t})$	$\log(\hat{\zeta}_{j,t})$	$\hat{\zeta}_{j,t}$
Capital intensity $_{t-1}$	0.153*** (0.055)	0.691** (0.326)	0.226* (0.132)
Financial friction $_{t-1}$	-0.469*** (0.035)	-0.934** (0.387)	-0.459*** (0.086)
Instability index $_{t-1}$	0.423*** (0.14)	-0.065 (0.107)	0.385* (0.225)
Industry uncertainty $_{t-1}$	-0.005 (0.014)	0.153** (0.077)	0.048 (0.794)
Industry age $_{t-1}$	-0.016 (0.029)	0.018 (0.034)	-0.004 (0.017)
Squared industry age $_{t-1}$	0.001*** (0.000)	0.000 (0.000)	-0.026 (0.035)
Industry growth $_{t-1}$	0.14 (0.346)	0.211 (0.266)	0.001*** (0.000)
R&D expenditure $_{t-1}$	- -	- -	- -
Constant	0.214 (0.166)	(dropped)	0.305 (0.228)
Standard errors	Drisc./Kray	Drisc./Kray	Robust
Time fixed effects	yes	yes	yes
Boxcox $\lambda = 0$	[0.045]	[0.045]	[0.045]
Log pseudo-likelihood	-	-	-471.692
Obs	429	429	429
Within R <sup>2</sup>	0.433	0.046	-

Note: Standard errors within brackets. Significance level (\*) refers to 90 percent, (\*\*) to 95 percent and (\*\*\*) to 99 percent. The table shows the results from running regression (3) for industries where a power law could not be rejected, where expenditures on R&D have been dropped. Except for the QMLE regression, which are run with robust standard errors, the regressions are run with Driscoll and Kray standard errors that are robust to very general forms of inter-temporal and cross-sectional dependence. In calculating the weighted least squares (WLS), I use analytical weights defined as the inverse of the standard error of the power law exponent from the first-stage regression in equation (3). These weights are computed using  $(\hat{\zeta}_{j,t})^{-1} (n/2)^{1/2}$  and give more weight to industries with small standard errors in the estimated power law exponent  $\hat{\zeta}_{j,t}$  (Ioannides et al., 2008). The FE estimator is computed using within transformation. In the QMLE regression, log-links have been specified. All variables have been lagged one time period to minimize potential simultaneity issues.

**Table A.2.1.** Correlation matrix for the independent variables

	Cap. int. <sup>a</sup>	Fin. fric.	Inst. ind.	Ind. age	Ind. age <sup>2</sup>	Ind. gr.	Ind. unc.	Ind. R&D
Cap. int. <sup>a</sup>	1.0000							
Fin. fric.	0.121**	1.000						
Inst. ind.	0.020	0.377**	1.000					
Ind. age	0.092**	-0.149**	-0.284**	1.000				
Ind. age <sup>2</sup>	0.165**	-0.293**	-0.213**	0.590**	1.000			
Ind. gr.	-0.006	-0.012	0.135**	-0.266**	-0.041	1.000		
Ind. unc.	0.107**	0.426**	0.329**	-0.384**	-0.280**	0.136**	1.000	
Ind. R&D	0.009	0.197**	0.097	0.124**	0.129**	-0.060	0.119*	1.000

Note: All of the variables are industry variables at the 3-digit NACE level. The correlations are pair wise correlations where (\*\*) indicates significance at the 95 percent level.

<sup>a</sup>The right tail of the capital intensity variable has been truncated at 95 percent at the firm level because of outliers.